Sixth Semester B.Sc., Degree Examinations

MAY/JUNE 2026

(NEP Syllabus)

MATHEMATICS

Paper: NSF 0570: LINEAR ALGEBRA AND FOURIER SERIES

Time: 2 hrs] [Max. Marks: 60

Instructions to Students:

1. ***The Students should legibly write Section number along with question numbers within the margin ONLY.***
2. ***The answer without Section number and question numbers will not be valued.***
3. ***Draw neat and labelled diagram wherever necessary.***
4. ***Use of Scientific non programmed calculator is permitted.***
5. ***Section – I multiple Choice Questions should be answered in the first page of main answer book and same questions shall not be answered repeatedly.***
6. ***Any identifications like marking, ticking, dots etc., in the options of MCQ (Section – I) on the question paper is strictly prohibited. Do not write anything (except register number) on the question paper.***

SECTION – I

Select the most appropriate answer from the options provided: $10 \times 1 = 10$

I - 1) Which of the following is a vector space?

a) $Z(Q)$ b) $Q(Z)$ c) $Q(R)$ d) $R(Q)$

I - 2) If $\{(1,-1,3), (1,2,-3), (k,0,1)\}$ is linearly dependent in R^3, then the value of k is

a) 2 b) -1 c) 1 d) -2

I - 3) A non empty subset B of a vector space $V(F)$ is said to be a basis of $V(F)$ if.

a) B is linearly independent set b) B spans V

c) Both (a) and (b) d) None of these

I - 4) A mapping $T:U \to V$ from the vectors space $U(F)$ into $V(F)$ is a linear transformation if

Contd..... 2

a) $T(\alpha+\beta)=T(\alpha)+T(\beta),\ \forall \alpha,\beta\in U$

b) $T(c\alpha)=cT(\alpha),\ \forall c\in F,\ \alpha\in U$

c) $T(c_1\alpha+c_2\beta)=c_1T(\alpha)+c_2T(\beta),\ \forall c_1, c_2\in F,\ \alpha,\beta\in U$

d) None of these

I - 5) The linear transformation $T:R^2\to R^2$ such that $f(1,0)=(1,1)$ and $f(0,1)=(-1,2)$ is

a) $f(x,y)=(x,y)$ b) $f(x,y)=(x-y,x+2y)$

c) $f(x,y)=(x+y,x-y)$ d) $f(x,y)=(x-y,x-2y)$

I - 6) If $T:V\to W$ is a linear transformation, then

a) $R(T)$ is a subspace of W b) $N(T)$ is a subspace of V

c) T is one-one if one only if $N(T)=\{0\}$ d) All of these answers are correct

I - 7) The norm of the vector $(2,-3,6)$ is

a) 1 b) 7 c) 49 d) -36

I - 8) The Schwartz inequality is

a) $|\langle x,y\rangle|\le\|x\|\|y\|$ b) $\|x+y\|\le\|x\|+\|y\|$

c) $|\langle x,y\rangle|\le\|x\|$ d) $|\langle x,y\rangle|\le\|y\|$

I - 9) The Fourier co-efficient b_n of the function $f(x)=x^2$, over the interval $(-\pi,\pi)$ is

a) 2 b) 1 c) -1 d) 0

I - 10) The Half Range cosine series of $f(x)$ over $(0,1)$ is

a) $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos n\pi x$ b) $f(x)=\sum_{n=1}^{\infty}a_n\cos n\pi x$

c) $f(x)=\frac{a_n}{2}+\sum_{n=1}^{\infty}\cos n\pi x$ d) $f(x)=a_0+\sum_{n=1}^{\infty}a_n\cos n\pi x$

Contd..... 3

SECTION – II

Answer any FIVE of the following: $5 \times 3 = 15$

II - 1) Prove that the intersection of any two subspaces of a vector space $V(F)$ is also a subspace of $V(F)$.

II - 2) Let $S=\{(1,-3,2),\ (2,4,1),\ (1,1,1)\}$ be a subset of $V_3(R)$. Show that the vectors $(3,-7,6)$ is in $L[S]$.

II - 3) A mapping $T:V_3(R)\to V_3(R)$ defined by $T(x_1,x_2,x_3)=(0,x_2,x_3)$ show that T is a linear transformation.

II - 4) Find the linear transformation $f:R^2\to R^2$ such that $f(1,1)=(0,1)$ and $f(-1,1)=(3,2)$.

II - 5) If $\alpha=(1,2,1)$ and $\beta=(2,-1,1)$, then find the angle between them.

II - 6) Show that $|\alpha+\beta|^2-|\alpha-\beta|^2=4\langle\alpha,\beta\rangle$

II - 7) Obtained the Fourier series of $f(x)=x^2, -\pi<x<\pi$ and $f(x+2\pi)=f(x)$.

II - 8) Represent the following function $f(x)$ as a Fourier cosine series, $f(x)=x,\ 0\le x\le L$

SECTION – III

Answer any THREE of the following: $3 \times 5 = 15$

III - 1) Prove that an ordered set $\{\alpha_1,\alpha_2,...,\alpha_n\}$ of non-zero n vectors of a vector space $V[F]$ with $\alpha_1\neq 0$, is linearly dependent if and only if one of the vectors say $\alpha_k\ (2\le k\le n)$ is a liner combination of its preceding ones.

III - 2) Find the dimension and basis of the subspace spanned by the vectors $(2,4,2)$, $(1,-1,0)$, $(1,2,1)$ and $(0,3,1)$ in $V_3(R)$.

III - 3) Find the matrix of the linear transformation $T:V_2(R)\to V_3(R)$ defined by $T(x,y)=(2y-x,\ y,\ 3y-3x)$ relative to the bases $B_1=\{(1,1),\ (-1,1)\}$ and $B_2=\{(1,1,1),\ (1,-1,1),\ (0,0,1)\}$

III - 4) Prove that every finite dimensional inner product space has orthonormal basis.

III - 5) Obtain the Fourier series of the function $f(x)=x-x^2$ in $-1<x<1$

Contd..... 4

SECTION – IV

Answer the following: $2 \times 10 = 20$

IV - 1) a) Prove that the necessary and sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a subspace of V are

i) $\forall \alpha, \beta \in W \Rightarrow \alpha + \beta \in W$

ii) $\forall c \in F, \alpha \in W \Rightarrow c\alpha \in W$

b) In $V_3(Z_3)$ how many vectors are spanned by $(1,2,1)$ and $(2,1,1)$. Find them (5 + 5)

OR

c) State and prove Rank – Nullity theorem.

d) Find the Range space and nullspace of the linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (y-x,\ y-z)$ (6 + 4)

IV - 2) a) Find the orthogonal basis and normal orthogonal basis of the subspace spanned by the vectors $\alpha_1 = (1,0,1)$, $\alpha_2 = (1,0,-1)$ and $\alpha_3 = (0,3,4)$ in $V_3(R)$ by Gram – Schmidt orthogonalization process.

b) In an Euclidean space if $|\alpha| = |\beta|$, then prove that $\alpha + \beta$ is orthogonal to $\alpha - \beta$. (6 + 4)

OR

c) Construct the Fourier Series for the function $f(x)$, given by $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.

d) Obtain the half range sine series for the function $f(x) = x,\ 0 \le x \le \pi$ (5 + 5)

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