Second Semester B.Sc., Degree Examinations

MAY/JUNE 2026

(SEP Scheme)

(New Syllabus 2024 – 25)

MATHEMATICS

PAPER SSB – 0240: ALGEBRA – II, CALCULUS – II AND DIFFERENTIAL EQUATIONS – I

Time: 3 hrs] *[Max. Marks: 80*

Instructions to Candidates:

1. ***The Students should legibly write Section number along with question numbers within the margin ONLY.***
2. ***The answer without Section number and question numbers will not be valued.***
3. ***Do not write anything (except register number) on the question paper.***

SECTION – A

I. Answer all the following questions: $10 \times 2 = 20$

1. a) Prove that the intersection of any two subgroups of a group G is a subgroup.

b) Define cyclic group.

c) If $f(x)=[x]$, where $[x]$ denotes the greatest integer not greater than x, examine $\lim_{x\to 1} f(x)$.

d) Verify the Rolle's theorem for the function $f(x)=x^2-6x+8$ in the interval $[2,4]$

e) Evaluate $\lim_{x\to 0}\dfrac{x-\sin x}{x^2}$

f) Find the order and degree of the differential equation $y=x\left(\dfrac{dy}{dx}\right)+\dfrac{1}{\left(\dfrac{dy}{dx}\right)}$

g) Solve $\left(x^2+1\right)\dfrac{dy}{dx}=1$.

h) Find the integrating factor of $\dfrac{dy}{dx}+\dfrac{1}{1+x^2}y=e^{\tan^{-1}x}$

i) Find all the asymptotes parallel to the x-axis of the curve $x^2y^2=a^2\left(x^2+y^2\right)$

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j) Find the envelope of the family of circles $(x-\alpha)^2+y^2=4\alpha$, where α is a parameter.

SECTION – B

II. Answer any SIX of the following questions: 6 × 5 = 30

2. Show that $G=\{0,1,2,3,4,5\}$ is an abelian group under addition modulo 6.
3. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $\forall a,b\in H \Rightarrow a*b^{-1}\in H$.
4. Discuss the continuity of $f(x)=\begin{cases}\dfrac{e^{1/x^2}}{1-e^{1/x^2}}, & for \quad x\neq 0\\ 1, & for \quad x=0\end{cases}$ at $x=0$.
5. Discuss the differentiability of the function $f(x)$ defined by
$$f(x)=\begin{cases}x^2\cos\left(1/x\right), & for \quad x\neq 0\\ 0, & for \quad x=0\end{cases} \text{ at } x=0.$$
6. Solve $(x-y)dy-(2x-y)dx=0$.
7. Solve $\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx+[x+\log x-x\sin y]dy=0$
8. Find the position and nature of the double points of the curve $x^3+y^3=3axy$.
9. Find all the asymptotes of the curve
$x^3-x^2y-xy^2+y^3+2x^2-4y^2+2xy+x+y+1=0$.

SECTION – C

III. Answer any THREE of the following questions: 3 × 10 = 30

10. a) Find all the right cosets of the subgroup $H=\{1,3,9\}$ of a group $G=\{1,2,3,4,5,6,7,8,9,10,11,12\}$ with respect to multiplication modulo 13.
    b) State and prove Fermat's theorem. (5 + 5)
11. a) Verify Lagrange's mean value theorem for $f(x)=x^2-3x-2$ in $[-2,3]$.
    b) Expand the function $\log_e(1+x)$ upto the term containing x^4 by Maclaurin's expansion. (5 + 5)

Contd..... 3

12. a) Solve $\frac{dy}{dx} + y\cot x = 4x \operatorname{cosec} x$, given that $y = 0$ when $x = \pi/2$.

    b) Solve $xy\,dx - \left(x^2 + 2y^2\right)dy = 0$ (5 + 5)

13. a) Find the envelope of the family of lines $y = mx + \sqrt{a^2m^2 + b^2}$, where m being the parameter .

    b) Find the position and nature of the double points of the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$ (5 + 5)

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